separate into ordinary differential equations along characteristic lines. Thus, along characteristic curves given by

$$dh/dt \;=\; \pm\, \rho a/\rho_0 \qquad (22)$$

the compatibility relations obtain:

$$dp \pm \rho a du \;=\; 0 \qquad (23)$$

These, combined with Eq. (17)

$$(\partial S/\partial t)_h \;=\; 0$$

can be solved by stepwise integration to yield a solution to the flow for given initial and boundary conditions.

Where the wave propagates into a constant state (simple wave) entropy is the same throughout the flow and Eq. (23) can be integrated to give the Riemann integral:

$$u - u_0 \;=\; \int dP/\rho a. \qquad (24)$$

This relation is frequently used in the analysis of experimental data for rarefaction or continuous compression waves. By measuring the propagation speed of pressure or mass velocity increments, Eq. (23) can be integrated numerically to yield pressure-velocity and pressure-density relations. Note that this method is a special case of the general method outlined in Section III.A. For this case, as for steady flow, the two phase velocities, c_p and c_u, are equal since $P \;=\; P(u)$.

The characteristics method breaks down at shock fronts not necessarily because the shock becomes a discontinuity but because the assumption of an equilibrium equation of state is untenable.

D. Relaxing Elastic-Plastic Solids[38]

For elastic-plastic relaxing solids the assumption of particle-isentropic flow is retained, or, more accurately, the effect of entropy changes on the pressure-density relation is ignored. Nonequilibrium states are allowed, however, by assuming that the plastic strain-rates may be time-dependent, and that the stresses depend only on the elastic strains. The plastic strains are assumed incompressible.

The relation to be added to the conservation relations is:

$$(\partial P/\partial t) - a^2(\partial \rho/\partial t) = -F(P,\rho) \tag{25}$$

where F is the plastic strain-rate multiplied by twice the shear modulus. The sound speed, a, is here the elastic longitudinal velocity, $a = \sqrt{(\lambda + 2\mu)/\rho}$. Combining this equation with Eqs. (7) and (8) in the same fashion as for isentropic flow in fluids yields the characteristic equations

$$c_+: \quad dP + \rho a du = -Fdt, \quad \text{on } dh/dt = \rho a/\rho_o$$

$$c_-: \quad dP - \rho a du = -Fdt, \quad \text{on } dh/dt = -\rho a/\rho_o$$

$$c_o: \quad dP - a^2 d\rho = -Fdt, \quad \text{on } dh/dt = 0$$

These equations have been applied to the solution of plane wave propagation in quartzite, which exhibits pronounced time-dependent effects.[38] They have also been applied to infer the plastic strain-rate at the elastic yield point in iron from measurements of the decay in elastic amplitude.[40]

At the elastic shock front, when an elastic precursor wave exists, the jump conditions can be applied to relate P and u through the relation

$$P_1 = \rho_o a u$$